FILE D

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Mathematics

Item Information and Scoring Guide Reference Sheet and Quantities of Items by Type

Item Information and Scoring Guide Reference Sheet

The following pages are designed to assist you in understanding how Maine Educational Assessment (MEA) items are scored. These pages contain the text for each item accompanied by the following information.

- **MC#:** the multiple-choice item position
- **Key:** the letter of the correct answer for the multiple-choice item
- **Learning Results:** the content standard, followed by the performance indicator, that the item measured

- **SA#:** the short-answer item position
- **Learning Results:** the content standard, followed by the performance indicator, that the item measured
- **Short-Answer Scoring Guide:** the two-point description used to determine the score
- **Training Notes:** in-depth descriptions or particular information used to determine the score

- **CR#:** the constructed-response item position
- **Learning Results:** the content standard, followed by the performance indicator, that the item measured
- **Constructed-Response Scoring Guide:** the four-point description used to determine the score
- **Training Notes:** in-depth descriptions or particular information used to determine the score

MAINE 2001–2002

Mathematics Grade 11

The table below shows the quantities of released items for each item type. Item information for all item types and scoring information (guides and training notes) for all short-answer and constructed-response items follow.

QUANTITIES OF ITEMS BY TYPE

MC	SA	CR
20	5	4

Items with Keys, Learning Results, Scoring Guides, Training Notes, and Student Responses

1. Mr. Willie put items costing the following amounts into his grocery cart.

$6.99	$2.36	$0.46	$11.63	$3.97	$0.52
$2.23	$1.32	$6.25	$2.49	$14.29	$2.76

Which of the following is the best estimate of the total price of these items?

A. between $30 and $40
B. between $40 and $50
C. between $50 and $60
D. between $60 and $70

MC#: 1
Key: C
Learning Results: B-1
Computation
B Students will understand and demonstrate computation skills. Students will be able to
1 use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

2. Which of the following is the best estimate of the sum of $\sqrt{26} + \sqrt{78}$?

   A. 10
   B. 14
   C. 52
   D. 104

MC#: 2
Key: B
Learning Results: B-1
Computation
B Students will understand and demonstrate computation skills. Students will be able to
1 use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

3. Monthly sales in February at Getco, Inc., were down 10% from January's sales. Monthly sales in March were up 10% over February's. How did sales in March compare with sales in January?

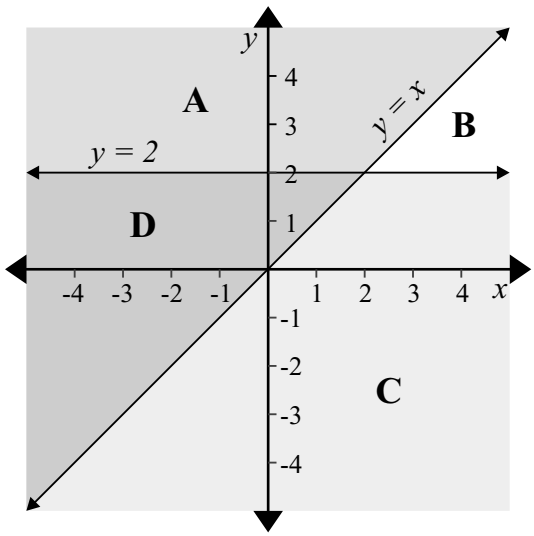
   A. Sales in March were higher.
   B. Sales in March were lower.
   C. Sales were the same in March and January.
   D. Not enough information is given to tell.

MC#: 3
Key: B
Learning Results: B-1
Computation
B Students will understand and demonstrate computation skills. Students will be able to
1 use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

4. Three business partners, Mr. Gray, Ms. Peterson, and Mr. Wu, share profits in the ratio of 2:3:5, respectively. Which of the following is the best estimate of Ms. Peterson's share of a profit of $24,925?

   A. $ 7,500
   B. $ 8,300
   C. $10,300
   D. $12,500

MC#: 4
Key: A
Learning Results: B-1
Computation
B Students will understand and demonstrate computation skills. Students will be able to
1 use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.



5. Which region of the graph represents the common solution set for both of the following inequalities?

$$y \geq x$$

$$y \leq 2$$

A. region A
B. region B
C. region C
D. region D

MC#: 5
Key: D
Learning Results: H-1
Algebra Concepts
H Students will understand and apply algebraic concepts. Students will be able to
1 use tables, graphs, and spreadsheets to interpret expressions, equations, and inequalities.

9. a. Find and write a real number between $\frac{5}{8}$ and $\frac{7}{9}$.

   b. Is it possible to find a real number between **any** two different real numbers? Justify your answer mathematically.

SA#: 9
Learning Results: A-1
Numbers and Number Sense
A Students will understand and demonstrate a sense of what numbers mean and how they are used.
Students will be able to
1 describe the structure of the real number system and identify its appropriate applications and limitations.

SHORT-ANSWER SCORING GUIDE

Score	Description
2	**Part a:** 1 point for any number between 5/8 and 7/9 in any form **Part b:** 1 point for any mathematically sound argument (e.g., average the two numbers; find a common denominator and use a numerator between the numerators of the new [equivalent] fractions)
1	Student answers either part a **or** part b correctly. There may be minor errors or omissions.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Short-Answer #9

- Probably the easiest way to determine whether a fraction is correct for part a is to convert it to a decimal and compare it with decimal equivalents of 5/8 and 7/9.
  (5/8 = 0.625 and 7/9 = 0.777…)
- Just writing, “Find the common denominator” is not sufficient.

Student Responses for Short-Answer #9 for Score Points 2 and 1

9. a. $\frac{b}{8}$

2

b. Yes, you can find a real number between any two other real numbers. There are an infinite number of possible decimal values.

9. a) $\frac{5}{8} = \frac{45}{72}$ $\frac{7}{9} = \frac{56}{72}$

$\frac{50}{72}$

b) yes

a,b The number is eq. $\frac{a+b}{2}$

2

9.

$\frac{5}{8}$ $\frac{6}{8}$ $\frac{3}{4}$ $\frac{7}{9}$

A) $\frac{3}{4} = .75$

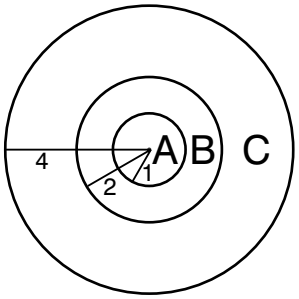
1

9. a. $\frac{5}{6}$ $\frac{7}{9} = \frac{45}{72}$ $\frac{50}{72}$ $\frac{56}{72}$

b. Yes because you can always use decimals Ex. 7 8 the number in between could be from 7.1 to 7.9.

1

10. Jennifer is designing a target game to play on the computer. Her target has three concentric circles with radii of lengths in the ratios of 1 to 2 to 4.



If a dot randomly generated by the computer lands within the target area, what is the probability that it will land in the area marked **A**? Show or describe how you found your answer.

SA#: 10
Learning Results: D-1
Probability
D Students will understand and apply concepts of probability. Students will be able to
1 find the probability of compound events and make predictions by applying probability theory.

SHORT-ANSWER SCORING GUIDE

Score	Description
2	• 1 point for correct answer of 1/16 or equivalent • 1 point for correct strategy; any errors are minor
1	Student either provides correct probability **or** correct strategy. There may be minor errors or omissions.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Short-Answer #10

Ratio of area A to area of entire circular: $\pi : 16\pi$ or 1:16
So probability is 1/16.

Do not penalize student for referring to, for example, the area of A as π rather than what it actually is—part of a ratio.

Student Responses for Short-Answer #10 for Score Points 2 and 1

10. **2**

Area of: A = π
B = 4π − π = 3π
C = 16π − 4π = 12π

Prob = $\frac{\pi}{16\pi}$ = $\frac{1}{16}$

10. **2**

$\frac{1}{16}$ ④ ③ Area C = $4^2\pi = 16\pi$
Area A = $1^2\pi = \pi$
A is $\frac{1}{16}$ of circle
$\frac{1}{16} = \frac{1\pi}{16\pi}$

10. **1**

Aof A = 1π Aof ABC = 16
Aof B = 4π ABC − C = 15
A of C = 16π
1 to 15 chance

10. **1**

1) π 2) 4π 3) 12π
The Prob. is
$\frac{1}{16}$ changes

11. The annual salaries of the employees of a small company are given below.

President: $120,000

Vice President: $75,000

Senior Professionals: $55,000, $52,000, $52,000, $52,000

Junior Professionals: $42,000, $42,000, $40,000, $39,000

Clerical Staff: $25,000, $20,000, $19,000

The statistical values for this data set are as follows:

- mean: $48,692
- median: $42,000
- mode: $52,000

If all three of these values are calculated again **without** the president's salary, which of the values—mean, median, or mode—would change the most? Justify your answer by comparing the effect of this change on all three statistics.

SA#: 11
Learning Results: C-1
Data Analysis and Statistics
C Students will understand and apply concepts of data analysis. Students will be able to
1 determine and evaluate the effect of variables on the results of data collection.

Score	Description
2 points	answering the mean together with complete justification of how the mean would change more than the median or mode
1 point	an answer showing some understanding of two of the statistics OR statement that mean would change without justification as to how or why
Blank	No response.

Training Notes for Short-Answer #11

Mean changes from $48,692 to $42,750, reduced by $5942.
Median is unchanged.
Mode is unchanged.
Student need not actually calculate mean as long as he/she shows that median and mode will not change and argues that mean must go down if highest number is eliminated.

Student Responses for Short-Answer #11 for Score Points 2 and 1

11. The mean would change the most.
w/out president:
Mean - 42,750
Median - 42,000
Mode - 52,000

2

11. the mean would change the most because the mode & median do not change

2

11. I think the mean would change the most, because that's like a total value, Median means, middle, so mean has the most change, would have

1

11. I think the mean would probably change the most, without the President's salary it would drop dramaticly. The mode would not change, the median might go down a little but not a lot,

1

12. Clem's balloon is 200 feet off the ground and rising at a rate of 5 feet per second. Mary's balloon is 100 feet off the ground and rising at a rate of 9 feet per second. In how many seconds will the two balloons be at the same height? Show how you found your answer.

SA#: 12
Learning Results: H-3
Algebra Concepts
H Students will understand and apply algebraic concepts. Students will be able to
3 formulate and solve equations and inequalities.

SHORT-ANSWER SCORING GUIDE

Score	Description
2	• 1 point for correct answer of 25 seconds (Accept number without "sec") • 1 point for correct strategy
1	Student answers one of above bullets correctly.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

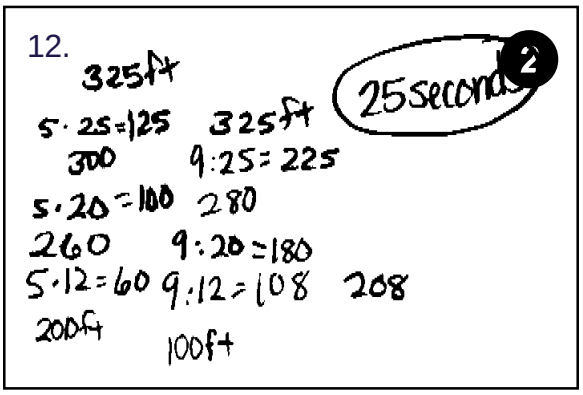
Training Notes for Short-Answer #12

Guess and check is adequate strategy if student indicates how he/she did it.
Algebraic solution:

$$200 + 5t = 100 + 9t$$
$$4t = 100$$
$$t = 25$$

Student Responses for Short-Answer #12 for Score Points 2 and 1

12.



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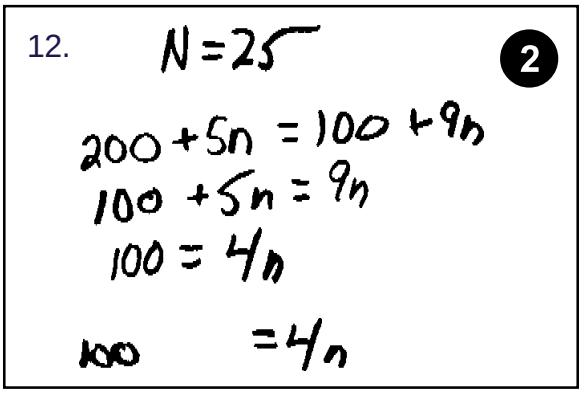

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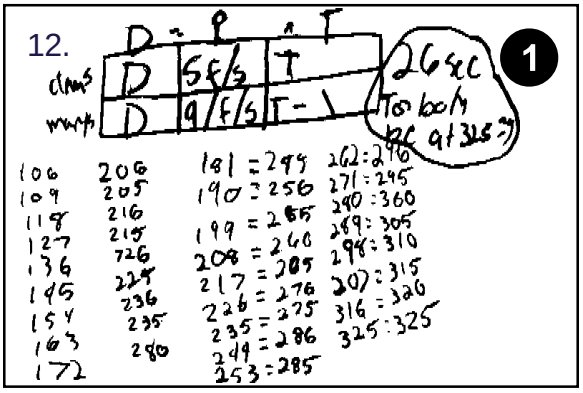

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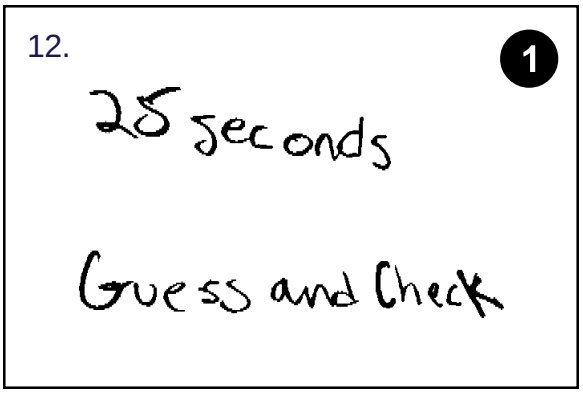
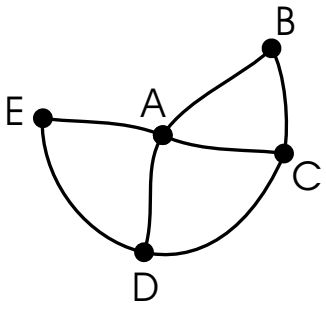




13. This network represents routes between cities on a map.

    a. Start at city C and find a route that would allow you to

    - drive over every route **exactly once** and
    - visit each city **at least** once.

    Record the route in your Student Response Booklet by listing the letters of the cities along your route in order.

    b. Starting at city E, can you find a route that meets the same conditions as in part a? If so, record the route. If not, explain why not.

SA#: 13
Learning Results: I-2
Discrete Mathematics
I Students will understand and apply concepts in discrete mathematics. Students will be able to
2 use networks to find solutions to problems.

SHORT-ANSWER SCORING GUIDE

Score	Description
2	**Part a:** 1 point for a route satisfying the conditions (e.g., CBACDEAD; CAEDABCD) **Part b:** 1 point for explanation or example demonstrating that it is impossible. There are two cities with three nodes so it is impossible to drive over every route to/from both cities exactly once.
1	Student answers either part a or part b correctly. There may be minor errors or omissions.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Student Responses for Short-Answer #13 for Score Points 2 and 1

13A. 1)C
2)B
3)A
4)C
5)D
6)E
7)A
8)D

B. No. There are two routes leaving part E and they cannot be traveled unless you go back over another. You need at least 3 routes leaving.

2

13.

a) C, B, A, C, D, A, E, D

b) E is not a proper starting area because it has an even number of paths.

2

13.

A. CDACBAED

B. No you can't because in order to go from E to D to A to E you would have to go back over a route you already crossed.

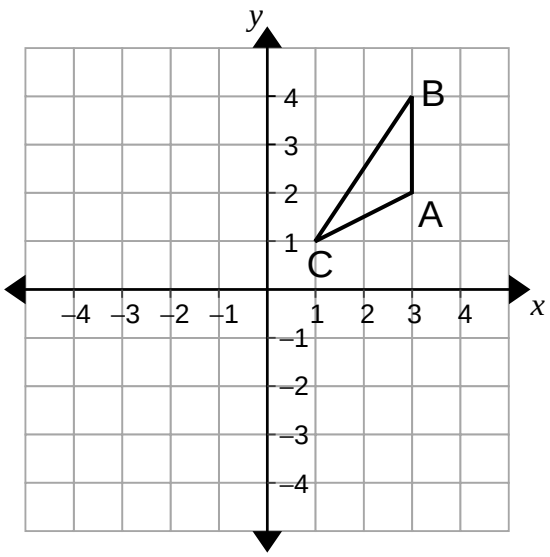
1

13.

C-A, A-B, B-C, C-d, d-E, E-A, A-D

B - No you cant theres not enough routes

1



16. Copy the triangle above onto the grid in your Student Response Booklet.

    a. Draw the reflection of ΔABC over the *x*-axis. Name the image you drew ΔA'B'C'.

    b. Draw the image of ΔA'B'C' rotated 180° counterclockwise around the origin. Name the image you drew A"B"C".

    c. Suppose point P on the interior of ΔABC had the coordinates (*p*,*q*). What would the coordinates of point P", the image of point P, be after the translations described in parts a and b?

CR#: 16
Learning Results: E-2
Geometry
E2 Students will understand and apply concepts from geometry. Students will be able to use inductive and deductive reasoning to explore and determine the properties of and relationships among geometric figures.

Score	Description
4	Student demonstrates a thorough understanding of coordinate plane transformations by accurately drawing and labeling the reflection and rotation of a figure and correctly determining the effect of transformations on coordinates of a given point.
3	Student demonstrates a general understanding of coordinate plane transformations by drawing and labeling the reflection and rotation of a figure and determining the effect of transformations on coordinates of a given point with only a minor error or omission.
2	Student demonstrates a basic understanding of coordinate plane transformations by correctly completing or using correct strategies to complete a significant portion of the required tasks.
1	Student demonstrates minimal understanding of coordinate plane transformations.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Constructed-Response #16

Part a (1 point)

- 1 point for correct reflection

Part b (1 point)

- 1 point for correct rotation

Part c (2 points)

- 1 point for each correct coordinate pair [part a: $(-p,q)$; part b: $(-p,-q)$]

OR

- 1 point for correct coordinate pairs not clearly labeled as to which is which

Notes:

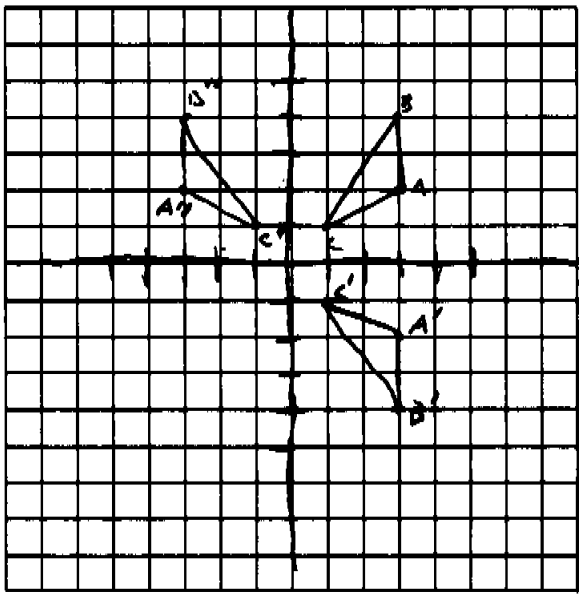
- if points C' and C" are located at the origin but not clearly labeled, do not penalize
- rotation should be judged relative to student's reflection

16.

a.b. look at graph

c. $P'' = (-P, q)$

4

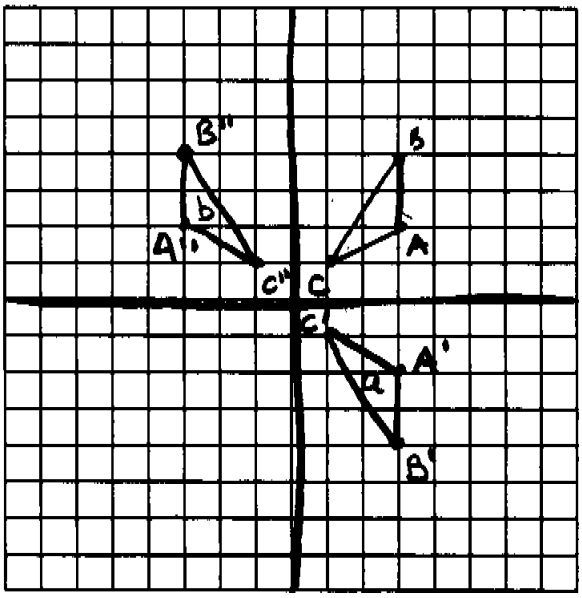


16.

c. part a: $(P, -q)$

part b: $(-P, q)$

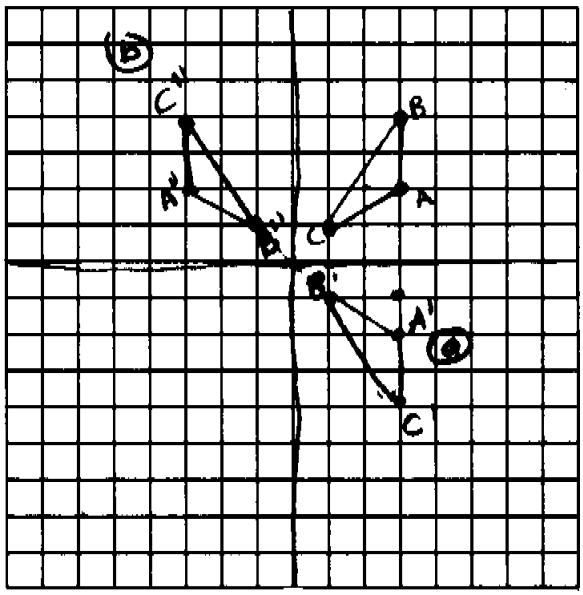
4



16.

3

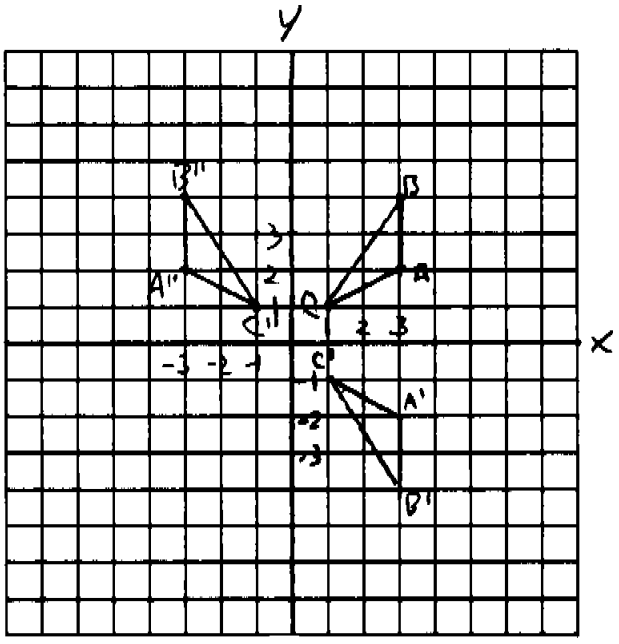
© point p,q in ABC would be point −p,q in A"B"C"



16.

3

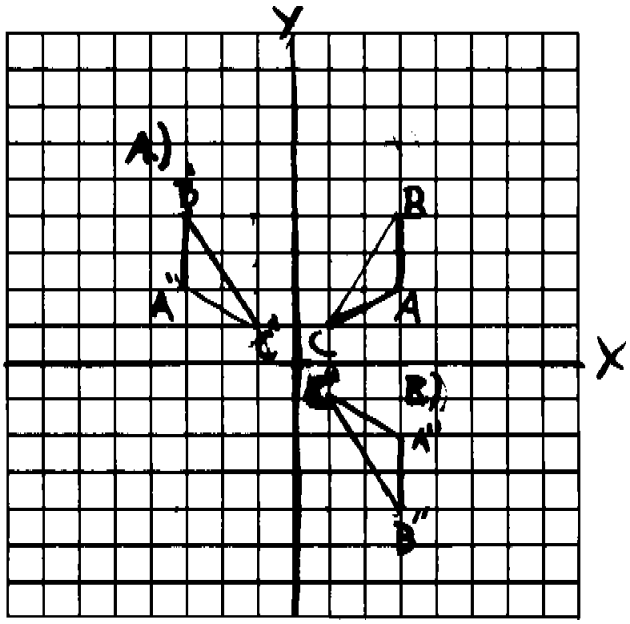
c) (−p, −q)



16.

2

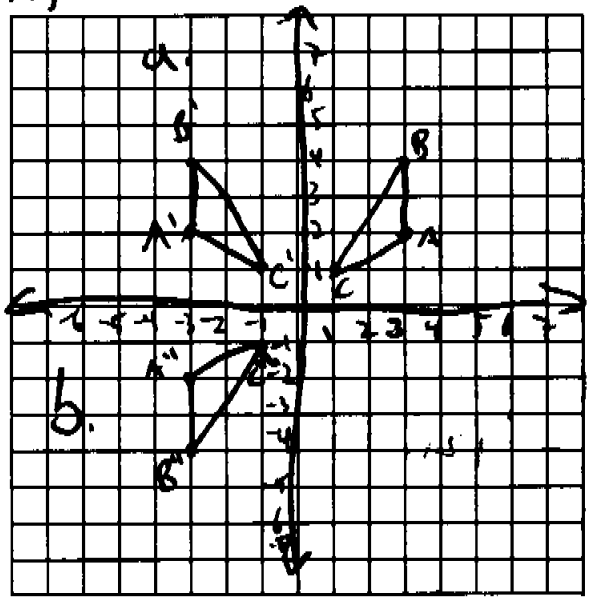
C.) $P'' = (-p, -q)$



16.

2

C. in part a, (p,q) would become (-p,q)
in part b, it would become (-p,-q)



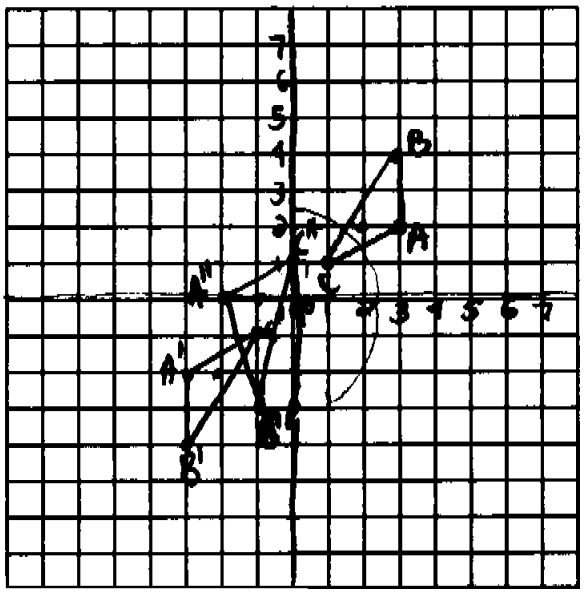
16.

1

ΔABC
(2,2)(3,4)(1,1)

	A		B		C	
	x	y				
A	3	2	3	4	1	1
A'	-3	-2	-3	-4	-1	-1
					1	0

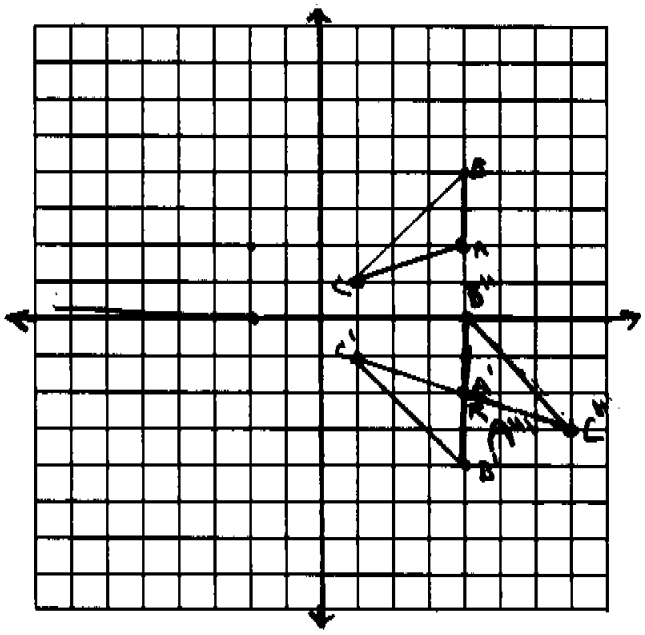
$P'' \rightarrow (-p, 0)$



16.

1

Point P'' would be at coordinates (p" q").



17. Carl needed to rent a car for one week. He collected the following information.

- Hillbrook Car Rentals charges $230/week plus 20¢ per mile.
- Bridge Car Rentals charges $210/week plus 25¢ per mile.

a. Write an equation for the cost, *c*, to rent from Hillbrook Car Rentals if Carl drives the car *x* miles.

b. Write an equation for the cost, *c*, to rent from Bridge Car Rentals if Carl drives the car *x* miles.

c. For what number of miles will it cost the same to rent a car from the two companies? Show or explain how you found your answer.

CR#: 17

Learning Results: G-3

Patterns, Relations, Functions

G Students will understand that mathematics is the science of patterns, relationships, and functions. Students will be able to

3 model phenomena using a variety of functions (linear, quadratic, exponential, trigonometric, etc.).

CONSTRUCTED-RESPONSE SCORING GUIDE

Score	Description
4	Student demonstrates thorough understanding of a real-life problem by creating correct linear models and using an appropriate strategy to obtain a correct solution to the problem.
3	Student demonstrates general understanding of a real-life problem by creating linear models and using an appropriate strategy to solve the problem, with only minor errors or omissions.
2	Student demonstrates basic understanding of a real-life problem by creating correct linear models or using correct strategies to solve a significant portion of the problem.
1	Student demonstrates minimal understanding of problem situation or algebraic modeling.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Constructed-Response #17

Score	Description
4	4 points
3	3 or 3½ points
2	2 or 2½ points
1	½ to 1½ points OR shows minimal understanding of writing equations or solving problem

Part a (1 point)

- 1 point for the correct equation, $c = 0.20x + 230$ (or equivalent)

OR

- ½ point for equation with inconsistent units ($c = 20x + 230$)

Part b (1 point)

- 1 point for the correct equation, $c = 0.25x + 210$ (or equivalent)

OR

- ½ point for equation with inconsistent units (e.g., $c = 25x + 210$)

Part c (2 points)

- 2 points for correct answer (400) or for the correct answer resulting from correctly solving student's equations in parts a and b simultaneously, with strategy shown or explanation given (see notes)

OR

- 1 point for correct strategy with arithmetic error **or** for correct answer with no work or explanation given

Notes

- Do not penalize student for not defining variables, for using a variable (other than *c*) to represent the number of weeks, or for using variable other than *c* for cost.
- Except for a 4-rating, accept correct expressions as well as equations for parts a and b.
- If student's equations in part a and b are correct, then the following strategy is correct:

$$0.20x + 230 = 0.25x + 210$$
$$20 = 0.05x$$
$$400 = x$$

- Also accept guess-and-check strategy **if** strategy is explained in detail with correct answer or with only minor computational errors. Student does not earn the point credit for just stating the strategy **guess and check.**

17.

4

a.) $C = 230 + .20x$
b.) $C = 210 + .25x$

cost = c
miles = x

c.) To find the number of miles it will take to cost the same amount to rent each rental car, simply set the two equations equal to each other.

$C = 230 + .2x$, $C = 210 + .25x$ $\quad C = C$

The drivers would pay the same price for the rental cars if they each drove the car exactly 400 miles each.

$230 + .2x = 210 + .25x$
$-210 - .2x \quad -210 - .2x$

$\frac{.20}{.05} = \frac{.05x}{.05}$

$400 = x$

* it would cost them both $310.00

17.

4

a) $c = .20x + 230$

b) $c = .25x + 210$

c) $x = 400$ miles

$c = .20(200) + 230 \quad c = 270$
$c = .25(200) + 210 \quad c = 260$

$c = .20(300) + 230 \quad c = 290$
$c = .25(300) + 210 \quad c = 285$

$c = .20(400) + 230 \quad c = 400$
$c = .25(400) + 210 \quad c = 400$

17.

3

a. $C = 230 + 20x$

b. $C = 210 + 25x$

$230 + 20x = 210 + 25x$

$20 = 5x$

$x = 4$ miles

Since in both equations you were solving for the cost, and you want to know when the two equal, set both equations equal to eachother.

17.

3

A.) $C = 230 + 0.20x$

B.) $C = 210 + 0.25x$

C.) 400 miles; guess and check

17.

2

a) $C = 230/\text{week} + .20x$

b) $C = 210/\text{week} + .25x$

17. ---

2

a. $c = 230 + .2x$

b. $c = 210 + .25x$

c. $210 + .25x = 230 + .2x$

$.05x = 220$

$x \text{ miles} = 440$

By setting the equations equal you are making the x values equal as well

17. 1

a. C = cost of Hillbrook
x = miles driven
$C = \$230.00 + 20x$

b. C = cost of Bridge's
x = miles driven
$C = \$210.00 + 25x$

Bridges $12.18
miles - 8.75
$C = 210.00 + 25x$
$-25 \quad -25$
$\frac{24.C}{24.} = \frac{210.00}{24.}$
$C = 8.75$

c. $C = 230.00 + 20x$
$-20 \quad -20x$
$\frac{-19C}{-19} = \frac{230.00}{-19}$
$C = -12.10$

Hillbrook $242.10 for 12.10 miles

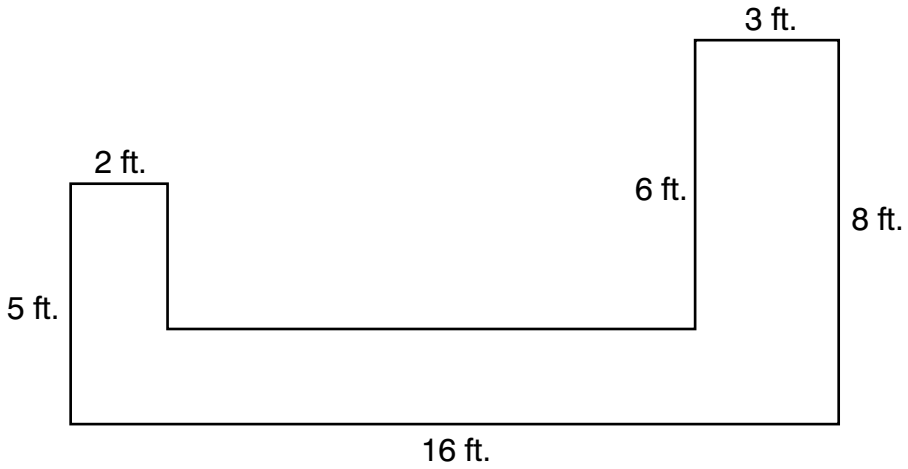
$-12.10 = 230.00 + 20x$
$-230.00 \quad -230.00$
$-242.1 = 20x \quad x = 12.105$

17. 1

a) $C = \$230 + \$20x$

b) C / x

c) I think if he rent his car from Bridge's it would be cheaper then from the other one.



19. Mr. Rilke is covering the floor of the hallway shown above with 1-foot square tiles. (All angles in the diagram are right angles.) How many tiles will he need?

    A. 50
    B. 55
    C. 56
    D. 66

MC#: 19
Key: C
Learning Results: F-2
Measurement
F 2 Students will understand and demonstrate measurement skills. Students will be able to derive and use formulas for area, surface area, and volume of many types of figures.

20. Three planets are in orbit around their sun. One planet's orbit takes 4 Earth years, another takes 36 Earth years, and the third takes 42 Earth years. If all three planets align at one position in their orbits this year, in how many Earth years will they **next** align in the same position?

A. 168
B. 252
C. 1,512
D. 6,048

MC#: 20
Key: B
Learning Results: B-1
Computation
B Students will understand and demonstrate computation skills. Students will be able to
1 use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

21. Jim calculated that a job will take 1.9 hours. If he starts the job at 12:04 P.M., at what time should it be completed?

A. 1:13 P.M.

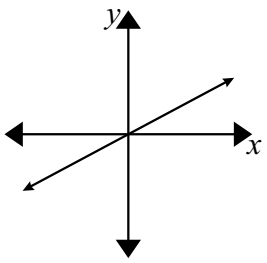
B. 1:54 P.M.

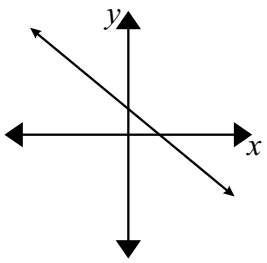
C. 1:58 P.M.

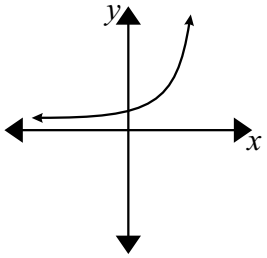
D. 2:34 P.M.

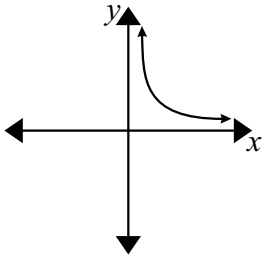
MC#: 21
Key: C
Learning Results: F-1
Measurement
F Students will understand and demonstrate measurement skills. Students will be able to
1 use measurement tools and units appropriately and recognize limitations in the precision of the measurement tools.

22. Which graph represents a relationship of direct variation?

A. 

B. 

C. 

D. 

MC#: 22
Key: A
Learning Results: H-2
Algebra Concepts
H Students will understand and apply algebraic concepts. Students will be able to
2 investigate concepts of variation by using equations, graphs, and data collection.

23. Susie is playing a game. At each turn she rolls two number cubes, each with sides numbered 1 through 6. Then she adds the numbers she rolled. What is the probability that on her next turn she will roll a sum of 4?

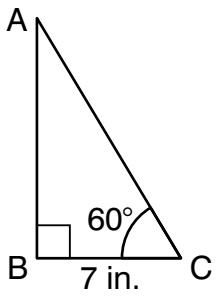
A. $\frac{2}{36}$

B. $\frac{3}{36}$

C. $\frac{4}{36}$

D. $\frac{6}{36}$

MC#: 23
Key: B
Learning Results: D-1
Probability
D Students will understand and apply concepts of probability. Students will be able to
1 find the probability of compound events and make predictions by applying probability theory.



24. To the nearest 0.1 inch, what is the length of $\overline{AB}$?

A. 3.5 in.
B. 9.9 in.
C. 12.1 in.
D. 14.0 in.

MC#: 24
Key: C
Learning Results: E-3
Geometry
E 3 Students will understand and apply concepts from geometry. Students will be able to apply trigonometry to problem situations involving triangles and periodic phenomena.

25. Imagine a monetary system in which the only paper bills available are worth $81, $27, $9, $3, and $1. What is the fewest number of bills that could be used to pay a bill of $275 exactly?

A. 5
B. 7
C. 9
D. 12

MC#: 25
Key: B
Learning Results: B-2
Computation
B Students will understand and demonstrate computation skills. Students will be able to
2 explain operations with number systems other than base ten.

26. Dave's Electronics is having a sale on radios. Each day the price of every radio will be reduced 5% from the previous day's price. If the price of a radio before the sale was \$50, which expression can be used to find its price on the nth day of the sale?

A. $50 - 0.05n$

B. $50 - 50n(0.95n)$

C. $50(0.05)^n$

D. $50(1 - 0.05)^n$

MC#: 26
Key: D
Learning Results: G-2
Patterns, Relations, Functions
G Students will understand that mathematics is the science of patterns, relationships, and functions. Students will be able to
2 translate and solve a real-life problem using symbolic language.

$y = (n - 1)(n)(n + 1)$
where n is a whole number greater than 1

27. Which of the following must be true about the value of y in the equation above?

    A. y is a multiple of 4
    B. y is a multiple of 5
    C. y is a multiple of 6
    D. y is a multiple of 7

MC#: 27
Key: C
Learning Results: H-4
Algebra Concepts
H 4 Students will understand and apply algebraic concepts. Students will be able to analyze and explain situations using symbolic representations.

28. Which of the following **cannot** be modeled by a linear equation?

A. the relationship between the length of a side of a cube and its volume

B. the relationship between the diameter of a circle and its circumference

C. the relationship between equivalent Celsius and Fahrenheit temperatures

D. the relationship between equivalent numbers of feet and meters

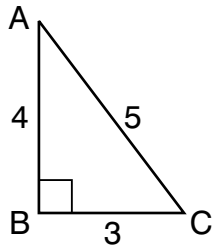
MC#: 28
Key: A
Learning Results: G-4
Patterns, Relations, Functions
G Students will understand that mathematics is the science of patterns, relationships, and functions. Students will be able to
4 identify a variety of situations explained by the same type of function.

29. A new Mainebucks lottery has been created. Each ticket contains three of the ten digits 0, 1, 2, 3, . . . , 9. (A digit can be used more than once on a ticket.) Each arrangement of digits has an equal chance of winning. If Mr. Watson buys one ticket, what are his chances of winning?

A. 1 out of 10
B. 1 out of 300
C. 1 out of 720
D. 1 out of 1000

MC#: 29
Key: D
Learning Results: D-1
Probability
D Students will understand and apply concepts of probability. Students will be able to
1 find the probability of compound events and make predictions by applying probability theory.

30. In ΔABC shown above, what is sin A equal to?



A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{4}{5}$

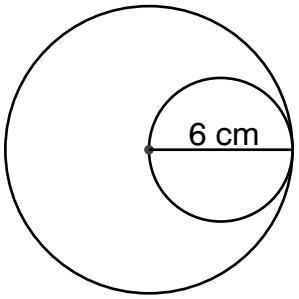
D. $\frac{5}{4}$

MC#: 30
Key: A
Learning Results: E-3
Geometry
E 3 Students will understand and apply concepts from geometry. Students will be able to apply trigonometry to problem situations involving triangles and periodic phenomena.

31. Five different books are placed side by side on a shelf. In how many different ways could they be arranged?

A. 5
B. 15
C. 25
D. 120

MC#: 31
Key: D
Learning Results: I-4
Discrete Mathematics
I 4 Students will understand and apply concepts in discrete mathematics. Students will be able to use matrices as tools to interpret and solve problems.

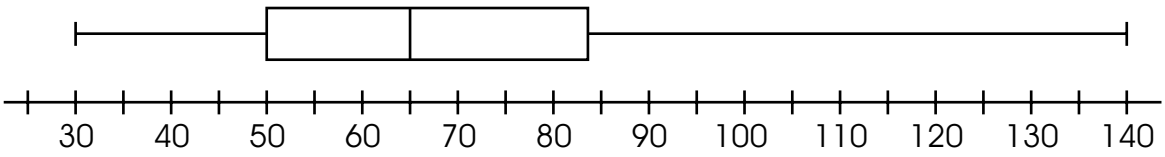


32. The larger circle has a radius of 6 centimeters. What is the ratio of the area of the smaller circle to the area of the larger circle?

A. 1:2
B. 1:3
C. 1:4
D. 1:6

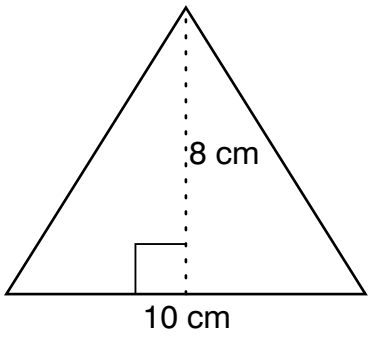
MC#: 32
Key: C
Learning Results: F-2
Measurement
F 2 Students will understand and demonstrate measurement skills. Students will be able to derive and use formulas for area, surface area, and volume of many types of figures.

33. What percent of the data represented by the box-and-whisker plot is greater than 65?



A. 32%
B. 50%
C. 68%
D. 75%

MC#: 33
Key: B
Learning Results: C-2
Data Analysis and Statistics
C Students will understand and apply concepts of data analysis. Students will be able to
2 predict and draw conclusions from charts, tables, and graphs that summarize data from practical situations.



40. a. Find the area of the triangle above.

    b. Suppose the base of the triangle above is increased by 10% and the height is decreased by 10%. What is the ratio of the area of the new triangle to that of the original triangle? Show or explain how you found your answer.

    c. Consider any triangle with a base, *b,* and a height, *h*. Suppose the base of the triangle is increased by 10% and the height is decreased by 10%. Will the ratio of the area of the new triangle to that of the original triangle remain the same for all values of *b* and *h*? Justify your answer mathematically.

CR#: 40
Learning Results: F-2
Measurement
F Students will understand and demonstrate measurement skills. Students will be able to
2 derive and use formulas for area, surface area, and volume of many types of figures.

CONSTRUCTED-RESPONSE SCORING GUIDE

Score	Description
4	Student demonstrates a thorough understanding of the area formula for triangles by correctly determining ratios of areas in both a particular and a general case.
3	Student demonstrates a general understanding of the area formula for triangles by determining ratios of areas in both a particular and a general case, with only minor errors and/or omissions.
2	Student demonstrates a basic understanding of the area formula for triangles by correctly completing or using correct strategies to complete a significant portion of the required tasks.
1	Student demonstrates minimal understanding of the area formula for triangles.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Constructed-Response #40

Score	Description
4	5 points
3	4 points OR 3 points with points from all three parts
2	2 or 3 points
1	1 point OR Student shows minimal understanding of area.

Part a (1 point)

- 1 point for correct area [40 (cm^2)]

Part b (2 points)

- 2 points for correct ratio (100:99 or equivelent) with work shown and/or justification

OR

- 1 point for correct answer with no work shown or justification **or** for correct strategy with arithmetic, or ratio conversion error

Part c (2 points)

- 2 points for correct ratio (100:99 or equivalent) with work shown and/or justification

OR

- 1 point for correct answer with no work shown or justification **or** for correct strategy with arithmetic or ratio conversion error

Solution Notes

Part a: area of original triangle: $\frac{1}{2} \times 8 \times 10 = 40$ (cm^2)

Part b: area of new triangle: $\frac{1}{2} \times 7.2 \times 11 = 39.6$ (cm^2)

ratio: 40:39.6 or 1:0.99 or 100:99

Part c: area of original triangle: $\frac{1}{2}bh$

area of new triangle: $\frac{1}{2}(1.1b \cdot 0.9h) = \frac{1}{2}(0.99bh)$

ratio is $\frac{1}{2}(bh) : \frac{1}{2}(0.99bh)$ or 1:0.99 or 100:99 or equivalent

- Student is not required to explicitly state areas. A student may notice that there is no need to deal with the (½)s and just drop them. However, if a student explicitly identifies an area as base times height, do not rate response as a 4; otherwise, do not penalize.
- Student may read entire problem, do part c, and then use it for part b. As long as student refers to the general case in his/her work on part b, he/she is eligible for a 4-rating.
- If student uses specific cases to "justify" answer to part c, at least two examples must explicitly demonstrate equivelent ratios to receive credit. Answer to part b may constitute one example.
- Student may reverse ratios throughout, as long as they are consistently in the same order, without penalty.

40.

4

a. $10 \times 8 \times \frac{1}{2} = 40$ 40cm^2

b. the base is increased by 10% — the height decreased by 10%

the base: 10 cm; $10 \times 0.1 = 1$; $10 + 1 = 11$ cm the new base

the height: 8 cm; $8 \times 0.1 = 0.8$; $8 - 0.8 = 7.2$ cm the new height

$11 \times 7.2 \times \frac{1}{2} = 39.6\ \text{cm}^2$ area of the new triangle

$39.6 : 40$

$9.9 : 10$

$99 : 100$

$99 : 100$

c. $1.1b$ the new base $0.9h$ the new height

$1.1b \times 0.9h \times \frac{1}{2} = 0.495bh$ the area of the new triangle

$\frac{1}{2}bh$ ... the area of the original triangle

$0.495bh = \frac{1}{2}bh$ (×2)

$0.99bh = bh$

$0.99 = 1$

$99 = 100$

Same ratio as the answer of b

the ratio of the area of the new triangle to that of the original triangle is same.

40.

4

(A) $\frac{10 \cdot 8}{2} = 40\ \text{cm}^2$

(B) 

$\frac{79.2}{2} = 39.6\ \text{cm}^2$ $\frac{39.6}{40} = \frac{99}{100}$

(C) $\frac{bh}{2}$ $\frac{(b \cdot 1.1)(h \cdot 0.9)}{2} = \frac{0.99bh}{2}$ $\frac{\frac{0.99bh}{2}}{\frac{bh}{2}} = 0.99$ or $\frac{99}{100}$

40.

3

A: $A = \frac{1}{2}bh = 5 \cdot 8 = 40\ cm^2$

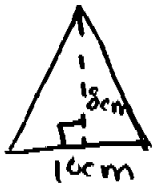
B: $A = (.9)(1.1b)(.9h) = .495bh$, ratio = 99:100 or $\frac{99}{100}$

C: yes, the Area formula ($A = \frac{1}{2} \cdot bh$) is constant for all triangles

$$\frac{\frac{1}{2}bh}{\frac{1}{2}(1.1b)(.9h)} = \frac{.5bh}{.495bb} = \frac{.5}{.495} = \frac{100}{99}, \text{ a } \frac{100}{99} \text{ is a constant (just like pi :)}$$

40.

3

A. 

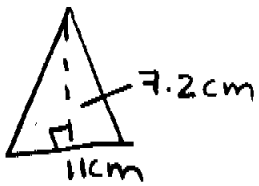
$\frac{1}{2}bh$

$\frac{1}{2}(b)(8)$

Area = 40cm

B $b = 10 + 10\%(10) = 10 + 1 = 11$

$h = 8 - 10\%(8) = 8 - .8 = 7.2$



The ratio of the new triangle to that of the original is $39.6\ cm : 40\ cm$ a 1% difference

$\frac{1}{2}(11)(7.2) = 39.6$

C. $\frac{h + (10(b/100))}{b - (10(b/100))}$ $10 - 10\%(1) = 9$

$= 8.8 = 39.6 : 40$

the ratio will stay the same for all values of b and h because they are all %'s of the original. One increases the other decreases by the same amount making the ratio continue being the same

40.

a) Area = $\frac{1}{2}bh$

Area = $\frac{1}{2} \cdot 10 \cdot 8$

Area = 40

b) Base = 10cm × .10 = 11

10cm + 1 = 11cm

Height = 8cm × .10 = .8

8cm − .8 = 7.2 cm

Area = $\frac{1}{2}(11)(7.2)$

Area = 39.6

39.6 × 5 = 198 ÷ 2 = 99

40 × 5 = 200 ÷ 2 = 100

100:99

old new

c) No because

2

40.

2

(a) Area of the triangle = 45cm

= area 45 cm

(b) new area of triangle

39.6 : 45

.88 : 1

= new area = 39.6 cm

(c) The ratio is going to be .5:1 because you are decreasing the base and they you cut that in half and multiply by the height to get your area.

40. **1**

a $A = \frac{1}{2}bH$

$A = \frac{1}{2}(10)(8)$

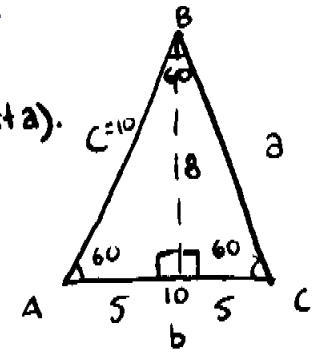
$A = 40$

b 8:10 or 4:5

7.2:11

C. no, the area would be off area is $\frac{1}{2}bh$ and if b and h cause the area would too.

40. **1**



Part a).

Each angle is 60° (180/3 = 60)

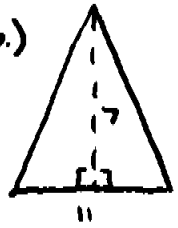
$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ $\sin 60\left[\frac{10}{\sin(60)} = \frac{c}{\sin(60)}\right]\sin 60$

$c = 10$

$A = \frac{1}{2}bh$ $A = \frac{1}{2}(10)(8)$

$A = 40\ cm^2$

Part b.)



New base = 11 cm
New height = 7cm

$10 \times .1 = 1$ $10 + 1 = 11$
$.8 - .1 = .7$ $.7 \times 10 = 7$

$A = \frac{1}{2}bh$ $A = \frac{1}{2}(11)(7)$

$A = 38.5$

The ratio of the two compared is that the New area is 1.5% less than the original.

Part c.) No, they will not.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(15)(5)$

$A = 37.5$

$A = \frac{1}{2}(16)(4)$

$A = 32$

32/37.5 = .853

Country	1997 Population (in millions)	Estimated 2002 Population (in millions)
China	1221.6	1272.3
India	967.6	1042.3
United States	268.0	279.5
Indonesia	209.8	225.6

41. This table shows the 1997 and estimated 2002 populations of the four most populous countries in the world. Jessica and Katy are using the data for a presentation of their world geography report.

    a. Jessica says that between 1997 and 2002 the population of China increased faster than that of the United States. Katy insists that the United States population increased more quickly. Explain how **each** of them can be correct. Justify your answer mathematically.

    b. Suppose the table was extended for additional 5-year periods (2007, 2012, 2017, etc.). If the population of the United States continues to grow at the percentage rate that is predicted in this table, after how many 5-year periods will the population reach 300 million? Show or explain how you found your answer.

CR#: 41
Learning Results: C-2
Data Analysis and Statistics
C Students will understand and apply concepts of data analysis. Students will be able to
2 predict and draw conclusions from charts, tables, and graphs that summarize data from practical situations.

Score	In this item, the student—
4	Student demonstrates a thorough understanding of data analysis by explaining the difference between percent increase and numerical increase and correctly applying percent change to predict future population growth.
3	Student demonstrates a general understanding of data analysis by explaining the difference between percent increase and numerical increase and applying percent change to predict future population growth, with only minor errors or omissions.
2	Student demonstrates some understanding of data analysis by explaining the difference between percent increase and gross increase or applying percent change to predict future population growth.
1	Student demonstrates minimal understanding of percent and/or numerical increase.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Training Notes for Constructed-Response #41

Score	Description
4	4 points
3	3 points
2	2 points
1	1 point OR demonstrates minimal understanding of percent and/or numerical increase

Part a (2 points)

2 points for correct explanation: China's increase was greater in terms of absolute numbers (50.7 million vs. 11.5 million) but the United States increase was greater in terms of percentage (4.29% vs. 4.15%) (may also show that U.S. population as percent or ratio of China population increased from 1997 to 2002).

OR

1 point for only one part of explanation correct or for correct strategy in both parts with arithmetic or minor procedural error(s) in each or for correct explanation without supporting arithmetic or percent values.

Part b (2 points)

2 points for correct conclusion that United States will reach 300 million in 2 periods following 2002 or 3 periods following 1997 (i.e., 2012), based on 4.29% increase per 5-year period (or percent for United States found in part a).

OR

1 point for correct strategy with error(s) in carrying out computations or interpreting results (e.g., doesn't find minimum number of periods) or for correct answer ("2 5-year periods") without work shown as support.

Note: Do not penalize in either part for rounding, or for incorrect rounding that does not affect the validity of the student's argument.

41. **4**

a. Jennifers statement that China is increasing faster is correct because it total population increased more. Katy, Statement that the US population is increasing more quickly is correct because it is growing at larger %.

China ↑ = 1272.3 - 1221.6 = 50.7 mill
US ↑ = 279.5 - 268.0 = 11.5 mill
} China is faster

or

China % ↑ = (1272.3/122.6) - 1 = 4.15%
US % ↑ = (279.5/268.0) - 1 = 4.29%
} US is faster

---

b. 2007 = 279.5 × 1.0429 = 291.49 mill

2012 = 291.49 × 1.0429 = 303.996 mill

After 2 more 5 year periods the US population will reach (and pass) 300 million.

41.

4

a)

$1272.3 - 1221.6 = 50.7$ $\quad 50.7 > 11.5$

$279.5 - 268 = 11.5$

However... $\frac{50.7}{1221.6} < \frac{11.5}{268}$

$\log_{10} 100 = 2$

$10^2 = 100$

b) After 2 five-year periods

$A(1+r)^t$ $\quad 300 = 279.5\left(1 + \frac{11.5}{268}\right)^t$

$1.0733... = \left(1 + \frac{11.5}{268}\right)^t$

$\log_{\left(1+\frac{11.5}{268}\right)} \frac{300}{279.5} = t$ $\quad \frac{\log \frac{300}{279.5}}{\log\left(1+\frac{11.5}{268}\right)} \approx 1.6846$

41.

3

a)

China's population increased by 50.7 million

The U.S.'s population increased 11.5 million

This makes Jennifer right

China's population increased by 3.98% $\left(\frac{50.7}{1272.3}\right)$

The U.S.'s population increased by 4.11% $\left(\frac{11.5}{279.5}\right)$

This made katy right

b)

1997	2002	2007	2012
~~268~~ 279 mil.	279.5 mil.	291.0 mil.	303 mil.
	*1.0411	*1.0411	*1.0411

41. 3

a. Jennifer is correct in that the number of millions of people gained total is greatest in China.

$$\begin{array}{r} 1272.3 \\ -1221.6 \\ \hline 50.7 \end{array}$$ million people gained.

However, Katy is right in that the U.S. has a higher percent gain than China.

China $\frac{50.7}{1221.6} = 0.041\ldots$ or about 4%.

U.S. $\begin{array}{r} 279.5 \\ -268.0 \\ \hline 11.5 \end{array}$ $\frac{11.5}{268} = 0.0429$ still about 4% but slightly higher than China

b. $300 = 268 + 0.0429(268)x$

$32 = 0.0429(268)x$

$32 = 11.4972x$

$x = 2.783$ 5 year periods

$268 + 0.0429(268)$

5 years 11.49 or 11.5 million

268
$+11.5$

$279.5 + 279.5(0.0429)$

5

11.86 million or 11.9 million at 2012

5, 5 year periods

one can only [illegible] in 5 more years 2017, population would exceed 300 million.

41.

2

China: 1272.3 − 1221.6 = 50.7

US: 279.5 − 268.0 = 11.5

a. Jennifer can be right because China's population increased by more individual people than the U.S.'s did. Katy can be correct because the U.S. rate of increase is higher than China's

$1221.6x = \frac{1272.3}{1221.6} \approx 1.0415$ China

$268.0x = \frac{279.5}{268} \approx 1.0429$ United States

b.

$268.0 + \frac{279.5}{268}x = 300$ (x-1)

$82 \cdot \frac{268}{279.5} = 30.6683 \approx 31$

$\frac{30.6683}{5} = 6$

After about 6 5-year period the United States' population will reach 300 million.

41.

2

Well China increased more in numbers of population. The united states increased more precent wise.

Chinas difference 50.7
US " " 11.5

percent (U.S)

$$\frac{279.5 - 268.0}{268.0} = \frac{11.5}{268.0}$$

$$= 4.3\%$$

percent (china)

$$\frac{50.7}{1221.6} = 4.2\%$$

41. 1

A- They both can't be correct. Jenifer is right, China is increasing by 10.14 million people a year, which was a total of 50.7 million people between 1997 and 2002. While the U.S. is only increasing by 2.5 million per year which is only a total of 11.5 million between 1997 and 2002.

B- Increase by 11.5 each 5 year period.

279.5 + 11.5 + 11.5 = 302.5

after 2 more 5 year periods

41. 1

a.) Jennifer can be correct because the actual rate at wich the population is growing is faster. Katy could be correct because more people could be coming to the united States increasing its population.

b.) 2 more 5-year time periods would be needed for the United States to reach 300 million.